1 P and Q are two points.

The coordinates of P are $(-1, 6)$

The coordinates of Q are $(5, -4)$

midpoint of PQ

Find an equation of the perpendicular bisector of PQ.
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

Finding midpoint of PQ :

$$\left(\frac{-1+5}{2}, \frac{6+(-4)}{2}\right) = (2, 1) \quad (1)$$

Finding gradient of line PQ :

$$m = \frac{(-4-6)}{(5-(-1))} = \frac{-10}{6} = \frac{-5}{3} \quad (1)$$

Finding gradient of perpendicular bisector :

$$m_{PB} = -\frac{1}{\frac{-5}{3}} = \frac{3}{5} \quad (1)$$

Finding equation of perpendicular bisector :

known value : point $(2,1)$ and gradient $= \frac{3}{5}$

$$y = mx + c$$

$$1 = \frac{3}{5}(2) + c$$

$$c = 1 - \frac{6}{5}$$

$$= -\frac{1}{5} \quad (1)$$

Equation : $y = \frac{3}{5}x - \frac{1}{5}$ (1)

$= 5y = 3x - 1$

$\therefore 3x - 5y - 1 = 0$

$3x - 5y - 1 = 0$ (1)

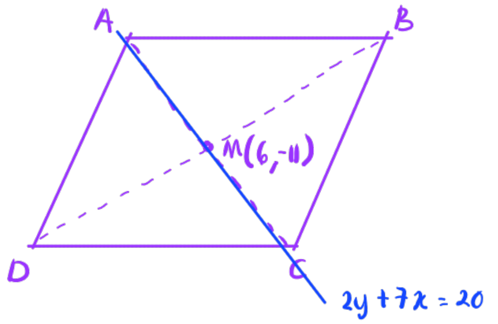
(Total for Question 1 is 6 marks)

2 *ABCD* is a rhombus.

The diagonals, *AC* and *BD*, intersect at the point *M*.
The coordinates of *M* are (6, −11)

The points *A* and *C* both lie on the line with equation $2y + 7x = 20$

Find the exact coordinates of the point where the line through *B* and *D* intersects the *y*-axis.



Equation of straight line AC :

$2y + 7x = 20$

$2y = 20 - 7x$

$y = \frac{-7x + 20}{2}$

$\therefore \quad y = -\frac{7}{2}x + 10$ where gradient $= -\frac{7}{2}$ (1)

Gradient of line BD $= \frac{-1}{m_{AC}} = \frac{2}{7}$ (1)

Equation of line BD :

$y = mx + c$

at M(6,-11) : $-11 = \frac{2}{7}(6) + c$

$c = -\frac{89}{7}$ (1)

$\therefore$ Line BD intersect y-axis at $\left(0, -\frac{89}{7}\right)$ (1)

(0 , $-\frac{89}{7}$)

(Total for Question 2 is 4 marks)

3 $\mathbf{L}_1$ and $\mathbf{L}_2$ are two straight lines.
The origin of the coordinate axes is O.

$\mathbf{L}_1$ has equation $5x + 10y = 8$
$\mathbf{L}_2$ is perpendicular to $\mathbf{L}_1$ and passes through the point with coordinates (8, 6)

$\mathbf{L}_2$ crosses the x-axis at the point A.
$\mathbf{L}_2$ intersects the straight line with equation $x = -3$ at the point B.

Find the area of triangle AOB.
Show your working clearly.

Equation of L_1 : $5x + 10y = 8$

$10y = -5x + 8$

$y = -\frac{1}{2}x + \frac{4}{5}$ in terms of $y = mx + c$

Gradient of L_1 : $-\frac{1}{2}$

Gradient of L_2 : 2 (1) $\boxed{m_{L_1} = \frac{-1}{m_{L_2}}}$

Equation of L_2 : $6 = 2(8) + c$

$c = -10$

$\therefore y = 2x - 10$ (1)

when L_2 crosses point A :

$y = 0$; $0 = 2x - 10$

$x = 5$ (1)

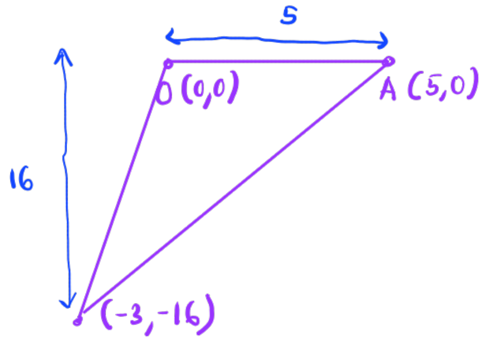
$\therefore L_2$ crosses point A at $(5, 0)$

when L_2 intersects at point B :

$x = -3$: $y = 2(-3) - 10$

$y = -16$

$\therefore L_2$ intersects at $(-3, -16)$



Area $= \frac{1}{2} \times 5 \times 16$ (1)

$= 40$ (1)

40

(Total for Question 3 is 5 marks)

4 (a) Write down an equation of a line that is parallel to the line with equation $y = 7 - 4x$

$y = -4x$ 1

....
(1)

The line **L** passes through the points with coordinates (–3, 1) and (2, –2)

(b) Find an equation of the line that is perpendicular to **L** and passes through the point with coordinates (–6, 4)
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

gradient of L : $\frac{-2-1}{2-(-3)} = -\frac{3}{5}$ 1

gradient of line ⊥ to L : $\frac{-1}{-\frac{3}{5}} = \frac{5}{3}$ 1

Equation of line ⊥ to L : $4 = \frac{5}{3}(-6) + c$

$c = 14$ 1

$\therefore \quad y = \frac{5}{3}x + 14$

$3y = 5x + 42$

$5x - 3y + 42 = 0$ 1

$5x - 3y + 42 = 0$

....
(4)

(Total for Question 4 is 5 marks)

5 The straight line **L** passes through the points (4, −1) and (6, 4)

The straight line **M** is perpendicular to **L** and intersects the y-axis at the point (0, 8)

Find the coordinates of the point where **M** intersects the x-axis.

gradient of line L : $\frac{4-(-1)}{6-4}$

$= \frac{5}{2}$ (1)

gradient of line M = $\frac{-1}{m_L}$

$= \frac{-1}{\frac{5}{2}} = -\frac{2}{5}$ (1)

Equation of line M : $y = -\frac{2}{5}x + 8$ intersects at y-axis (0,8)

when M intersects x-axis, $y = 0$

$0 = -\frac{2}{5}x + 8$

$\frac{2}{5}x = 8$

$x = \frac{8 \times 5}{2}$

$= 20$ (1)

∴ M intersects x-axis at (20,0) (1)

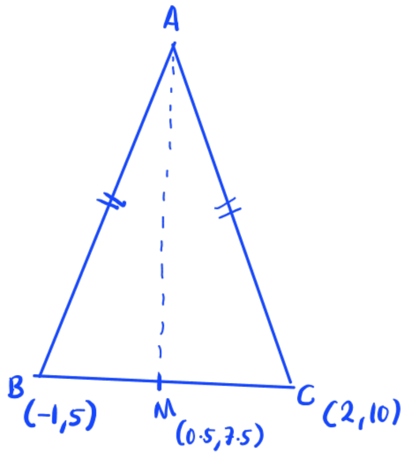
(..........20.........., ...0..........)

(Total for Question 5 is 4 marks)

6 *ABC* is an isosceles triangle with *AB* = *AC*.

B is the point with coordinates (–1, 5)
C is the point with coordinates (2, 10)
M is the midpoint of *BC*.

Find an equation of the line through the points *A* and *M*.
Give your answer in the form $py + qx = r$ where *p*, *q* and *r* are integers.



midpoint of BC $= \left(\frac{2+(-1)}{2}, \frac{10+5}{2}\right)$

$= (0.5, 7.5)$ (1)

gradient of line BC : $\frac{10-5}{2-(-1)}$

$= \frac{5}{3}$ (1)

gradient of line MA $= \frac{-1}{m_{BC}}$

$= -\frac{3}{5}$ (1)

Equation of line MA = $7.5 = -\frac{3}{5}(0.5) + c$

$c = 7.5 + 0.3$

$= \frac{39}{5}$ (1)

$y = -\frac{3}{5}x + \frac{39}{5}$

$5y = -3x + 39$

$5y + 3x = 39$ (1)

$5y + 3x = 39$

....

(Total for Question 6 is 5 marks)

The straight line **L** has equation $y = -4x + 5$

7 (b) Write down the gradient of a straight line that is perpendicular to **L**.

perpendicular lines mean $m_1 m_2 = -1$

$m_1 = -4$

$-4(m_2) = -1$

$m_2 = \frac{1}{4}$

$\frac{1}{4}$ (1)

....

(1)

(Total for Question 7 is 1 marks)

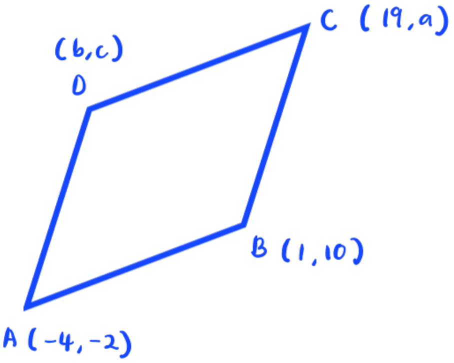
8 A rectangle *ABCD* is to be drawn on a centimetre grid such that

A has coordinates (–4, –2)
B has coordinates (1, 10)
C has coordinates (19, *a*)
D has coordinates (*b*, *c*)

(a) Work out the value of *a*, the value of *b* and the value of *c*.

Difference in x-axis between AB = $1-(-4) = 5$

That means $b = 19-5$
$b = 14$ (1)



Gradient AB $= \dfrac{10-(-2)}{1-(-4)}$

$= \dfrac{12}{5}$ (1)

Gradient BC $= \dfrac{a-10}{19-1}$

$= \dfrac{a-10}{18}$

$\dfrac{12}{5} \times \dfrac{a-10}{18} = -1$

perpendicular lines :

$m_1 m_2 = -1$

$\dfrac{12(a-10)}{90} = -1$

$12a - 120 = -90$

$12a = 30$

$a = 2.5$ (1)

Difference in y-axis between AB $= 10-(-2)$
$= 12$

$c = 2.5 - 12 = -9.5$ (1)

a = 2.5

b = 14

c = −9.5

(4)

(b) Calculate the perimeter, in centimetres, of rectangle *ABCD*.

$$AB = \sqrt{(1-(-4))^2 + (10-(-2))^2}$$

$$= \sqrt{5^2 + 12^2}$$

$$= 13 \quad (1)$$

$$BC = \sqrt{(19-1)^2 + (2.5-10)^2}$$

$$= 19.5 \quad (1)$$

$$\text{Perimeter} = 2(13) + 2(19.5)$$

$$= 65 \text{ cm} \quad (1)$$

65 cm

(3)

(Total for Question 8 is 7 marks)

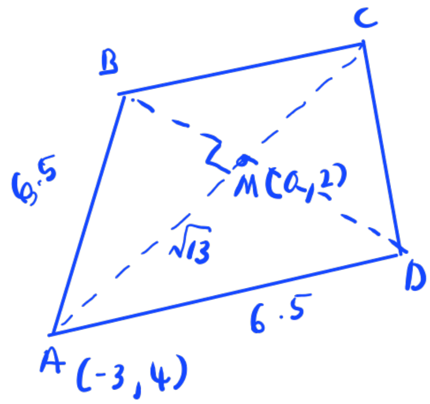
9 *ABCD* is a kite, with diagonals *AC* and *BD*, drawn on a centimetre square grid, with a scale of 1 cm for 1 unit on each axis.

A is the point with coordinates (−3, 4)

The diagonals of the kite intersect at the point *M* with coordinates (0, 2)

Given that *AB* = *AD* = 6.5 cm and the *x* coordinate of *B* is positive,

find the coordinates of the points *B* and *D*.



$m_{AM} = \frac{4-2}{-3} = -\frac{2}{3}$ (1)

$m_{BD} = \frac{3}{2}$

Equation of line BD : $y - 2 = \frac{3}{2}x$

$y = \frac{3}{2}x + 2$ (1)

$AM = \sqrt{(-3-0)^2 + (4-2)^2} = \sqrt{13}$

$BM = \sqrt{(x-0)^2 + (y-2)^2} = \sqrt{x^2 + (y-2)^2}$ (1)

$AB^2 = AM^2 + BM^2$

$(6.5)^2 = 13 + x^2 + (y-2)^2$ (1)

$\frac{117}{4} = x^2 + (y-2)^2$

$\frac{117}{4} = x^2 + \left(\frac{3}{2}x\right)^2$ (1)

$\frac{117}{4} = \frac{13}{4}x^2$ (1)

$x^2 = \frac{117}{13} = 9$

$x = \pm 3$

$x = 3, \quad y = 6.5$

$x = -3, \quad y = -2.5$

1

(......3......, ...6.5......)

(......-3......, ...-2.5......)

(Total for Question 9 is 7 marks)

10 G is the point on the curve with equation $y = 8x^2 - 14x - 6$ where the gradient is 10
The straight line **Q** passes through the point G and is perpendicular to the tangent at G

Find an equation for **Q**
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

gradient, $\frac{dy}{dx} = 16x - 14$ (1)

$16x - 14 = 10$ (1)

$x = \frac{24}{16} = 1.5$

$y = 8(1.5)^2 - 14(1.5) - 6$

$= -9$ (1)

$G(1.5, -9)$

$m_Q = -\frac{1}{10}$

$-9 = -\frac{1}{10}\left(\frac{3}{2}\right) + c$

$-9 + \frac{3}{20} = c$

$\frac{-177}{20} = c$ (1)

$y = -\frac{1}{10}x - \frac{177}{20}$

$20y = -2x - 177$

$2x + 20y + 177 = 0$ (1)

$2x + 20y + 177 = 0$

(Total for Question 10 is 5 marks)

11 *ABCD* is a trapezium with *AB* parallel to *DC*

A is the point with coordinates (–4, 6)
B is the point with coordinates (2, 3)
D is the point with coordinates (–1, 8)

The trapezium has one line of symmetry.
The line of symmetry intersects *CD* at the point *E*

Work out the coordinates of the point *E*

midpoint AB : $\left(\frac{-4+2}{2}, \frac{6+3}{2}\right)$

$= (-1, 4.5)$ (1)

gradient AB : $\frac{6-3}{-4-2} = -\frac{1}{2}$ (1)

gradient of symmetry line = 2 (1)

DC: $y - 8 = -0.5(x - (-1))$

$y - 8 = -0.5x - 0.5$

$y = -0.5x + 7.5$ — (1) (1)

symmetry line : $y - 4.5 = 2(x - (-1))$

$y - 4.5 = 2x + 2$

$y = 2x + 6.5$ — (2) (1)

$2x + 6.5 = -0.5x + 7.5$

$2.5x = 1$

$x = \frac{1}{2.5} = 0.4$

$y = 2(0.4) + 6.5 = 7.3$

1

(......0.4......, ...7.3......)

(Total for Question 11 is 6 marks)

12 *ABCD* is a kite.

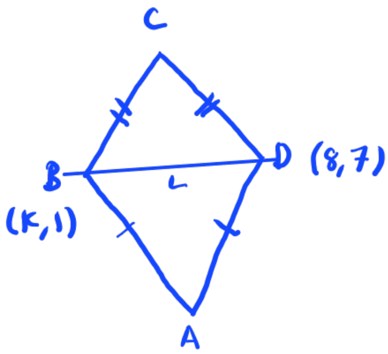
AB = *AD* and *CB* = *CD*

The point *B* has coordinates (k, 1) where k is a negative constant.
The point *D* has coordinates (8, 7)

The straight line **L** passes through the points *B* and *D*

The straight line **L** is parallel to the line with equation $5y - 3x = 6$

Find an equation of *AC*
Give your answer in the form $px + qy = r$ where p, q and r are integers.
Show your working clearly.



gradient of L : $5y = 3x + 6$

$y = \frac{3}{5}x + \frac{6}{5}$ 1

$m_L = \frac{3}{5}$

$\frac{3}{5} = \frac{7-1}{8-k}$

$24 - 3k = 35 - 5$

$3k = 24 - 30$

$3k = -6$

$k = -2$ 1

midpoint of BD : $\left(\frac{8+(-2)}{2}, \frac{7+1}{2}\right)$

$= (3, 4)$ 1

gradient of AC : $-\frac{5}{3}$ 1

Equation of AC : $y - 4 = -\frac{5}{3}(x - 3)$ 1

$3y - 12 = -5x + 15$

$3y = -5x + 27$

$5x + 3y = 27$ 1

$5x + 3y = 27$

(Total for Question 12 is 6 marks)

13 The straight line with equation $y - 2x = 7$ is the perpendicular bisector of the line AB where A is the point with coordinates $(j, 7)$ and B is the point with coordinates $(6, k)$

Find the coordinates of the midpoint of the line AB
Show clear algebraic working.

$y = 2x + 7$

$m = 2$

$m_{AB} = -\frac{1}{2}$ (1)

$-\frac{1}{2} = \frac{k-7}{6-j}$ (1)

$-6 + j = 2k - 14$

$2k - j = 8$ — (1)

midpoint of AB : $\left(\frac{j+6}{2}, \frac{7+k}{2}\right)$ (1)

$\frac{7+k}{2} = \cancel{2}\left(\frac{j+6}{\cancel{2}}\right) + 7$

(1)

$7 + k = 2j + 12 + 14$

$k = 2j + 19$ — (2)

substitute (2) into (1) :

$2(2j + 19) - j = 8$

$4j + 38 - j = 8$

$3j = -30$

$j = -10$ (1)

$k = 2(-10) + 19$

$= -1$

midpoint of AB : $\left(\frac{-10+6}{2}, \frac{7-1}{2}\right)$

$= (-2, 3)$ (1)

(-2 , 3)

(Total for Question 13 is 6 marks)

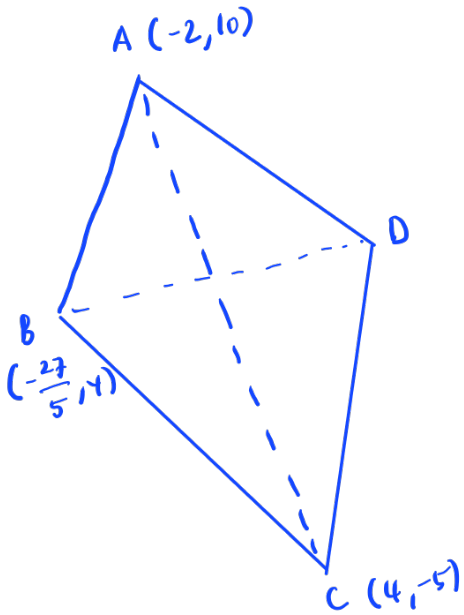
14 $ABCD$ is a kite with $AB = AD$ and $CB = CD$

A is the point with coordinates $(-2, 10)$

B is the point with coordinates $\left(-\frac{27}{5}, 4\right)$

C is the point with coordinates $(4, -5)$

Work out the coordinates of D



gradient AC : $\frac{-5-10}{4-(-2)} = \frac{-15}{6} = -\frac{5}{2}$ (1)

equation of AC : $10 = -\frac{5}{2}(-2) + c$

$c = 10 - 5 = 5$

$\therefore \quad y = -\frac{5}{2}x + 5$ (1)

gradient BD : $\frac{2}{5}$

equation of BD : $4 = \frac{2}{5}\left(-\frac{27}{5}\right) + c$

$4 = -\frac{54}{25} + c$

$c = \frac{154}{25}$

$\therefore \quad y = \frac{2}{5}x + \frac{154}{25}$ (1)

$-\frac{5}{2}x + 5 = \frac{2}{5}x + \frac{154}{25}$ (1)

$\frac{2}{5}x + \frac{5}{2}x = 5 - \frac{154}{25}$

$2.9x = -\frac{29}{25}$ (1)

$x = \frac{-10}{25} = -\frac{2}{5}$, $y = -\frac{5}{2}\left(-\frac{2}{5}\right) + 5 = 6$

intersection between AC and BD is $\left(-\frac{2}{5}, 6\right)$

$$\left(-\frac{2}{5}, 6\right) = \left(\frac{-\frac{27}{5} + x_D}{2}, \frac{4 + y_D}{2}\right)$$

x_D : $-\frac{4}{5} + \frac{27}{5} = \frac{23}{5}$ (1)

y_D : $12 - 4 = 8$

($\frac{23}{5}$, 8)

(Total for Question 14 is 6 marks)